UNSTEADY MOISTURE TRANSFER IN CAPILLARY-POROUS SUBSTANCES

S. A. Tanaeva

Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 5, pp. 589–593, 1965
UDC 532.63+532.546

A derivation and solution are given for the differential equation of unsteady moisture transfer in capillary-porous substances, allowing for relaxation phenomena. Results are given of an experimental verification of the solution obtained.

Mass transfer in capillary-porous substances is known to obey Fick's law

$$i = -D\frac{\partial \rho}{\partial n}. \tag{1}$$

However, if the liquid is regarded as a viscoelastic substance, then elastic strains arise in the liquid due to friction forces, during its flow through the capillary-porous medium, and these strains relax with some period τ_r. In that case (1) will contain an additional term associated with the relaxation processes

$$i = -D\frac{\partial \rho}{\partial n} - \tau_r\frac{\partial i}{\partial \tau}, \tag{2}$$

where $\tau_r = D/w^2$.

The diffusion coefficient is proportional to the rate of propagation of the isoconcentration surface. In fact, if the equation of the isoconcentration surface is $\rho(x, y, z\tau)$ = const, then the total differential

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial \rho}{\partial n} w = 0. \tag{3}$$

It follows from the differential equation of mass conduction that

$$w = -D\frac{\nabla^2 \rho}{\nabla \rho} = AD.$$

The quantity $A = \nabla^2\rho/\nabla\rho$ is a ratio of differential parameters, is invariant with respect to a linear group of transformations, and has dimension M.

Equation (1) is evidently a special case of (2) when $w \to \infty$. If $w \to 0$, Eq. (2) may be written in the form

$$i = -\tau_r\frac{\partial i}{\partial \tau}.$$

It is interesting to note that a similar picture occurs also in investigation of viscoelastic flow of a liquid described by the equation [3]

$$P = \eta\dot{\varepsilon} - \tau_r\frac{dP}{d\tau},$$

where $\tau_r = \eta/G$.

In the case of a solid ($\eta \to \infty$) we obtain Hooke's well-known law

$$P = G\varepsilon.$$

On the basis of the above mass transfer equation, we shall derive the differential equation of moisture conduction. Differentiating (2) with respect to x:

$$\frac{\partial i}{\partial x} = -D\frac{\partial^2 \rho}{\partial x^2} - \tau_r\frac{\partial^2 i}{\partial \tau\, \partial x}.$$

Using the fact [1,2] that $\partial\rho/\partial\tau = -\text{div}\, i$, we have

$$\frac{\partial \rho}{\partial \tau} = D\frac{\partial^2 \rho}{\partial x^2} - \tau_r\frac{\partial^2 \rho}{\partial \tau^2}. \tag{4}$$

Knowing that the concentrations of moisture and humidity are related by

$$\rho = \gamma_0 U,$$

we may write (4) in the form

$$\frac{\partial U}{\partial \tau} = D\frac{\partial^2 U}{\partial x^2} - \tau_r\frac{\partial^2 U}{\partial \tau^2}. \tag{5}$$

Granulometric Composition of the Substances Used

Material	Granulometric composition, % in size ranges, mm								Specific gravity
	2—1	1—0.5	0.5—0.25	0.25—0.1	0.1—0.05	0.05—0.01	0.01—0.005	<0.005	
Sand 1	0.6	9.5	69.8	18.0	0.90	0.50	0 70		2.66
Sand 2	0.3	0.20	5.0	79.2	13.3	0.4	1.6		2.61
Clay			0.30	0.50	8.7	33.5	14.9	42.10	2.82

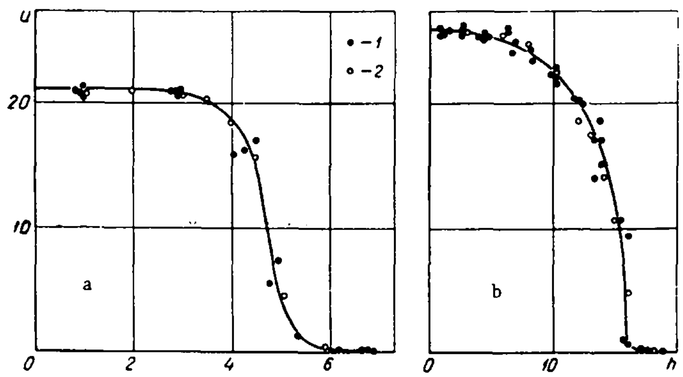

Fig. 1. Dependence of moisture content (U, %) on the height of capillary rise (h, cm) in Sand 1 (a) and Sand 2 (b): 1) experimental values: 2) according to (13).

Let us put $1/\tau_r = 2h$, $w = a$. Then (5) takes the form

$$\frac{\partial^2 U}{\partial \tau^2} + 2h\frac{\partial U}{\partial \tau} = a^2\frac{\partial^2 U}{\partial x^2}. \qquad (6)$$

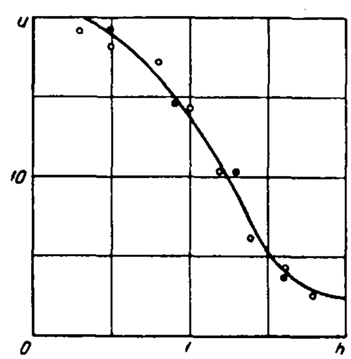

Fig. 2. Dependence of U on h in clay: 1,2) as in Fig. 1.

We shall solve (6) with the following initial and boundary conditions:

$$U(x,\,0) = \varphi_1(x); \quad \frac{\partial U(x,\,0)}{\partial \tau} = \varphi_2(x),$$

$$\frac{\partial U(0,\,\tau)}{\partial x} = 0, \qquad (7)$$

$$U(l,\,\tau) = U_c, \quad U_c = \text{const}. \qquad (8)$$

We shall look for particular solutions of (6) which are not identically equal to zero, in the form (4)

$$U(x,\,\tau) = X(x)T(\tau). \qquad (9)$$

Substituting (9) into (6), we obtain the two ordinary differential equations

$$T'' + 2hT' + a^2\lambda T = 0, \qquad (10)$$

$$X'' + \lambda X = 0, \qquad (11)$$

where λ is some constant.

In order to obtain-trivial solutions of Eq. (6) in the form of (9), satisfying boundary conditions (7) and (8), it is necessary that the function X(x) shall satisfy the boundary conditions

$$X'(0) = 0, \quad X(l) = U_c. \qquad (12)$$

The solution of (11) for $\lambda > 0$ and for the boundary conditions (12) has the form

$$X(x) = \cos\sqrt{\lambda}\,x,$$

where

$$\sqrt{\lambda} = (2n-1)\pi/2l.$$

The solution of (10) when $h^2 < a^2\lambda$ has the form

$$T = (a_n\cos k\tau + b_n\sin k\tau)\exp(-h\tau),$$

where

$$k = \sqrt{a^2\lambda - h^2}.$$

Then the solution of (6) may be written in the form

$$U(x,\,\tau) = U_c - \exp(-h\tau)\times$$
$$\times\sum_{n=1}^{\infty}(A_n\cos k\tau + B_n\sin k\tau)\cos\mu\frac{x}{l}, \qquad (13)$$

where

$$\mu = (2n-1)\pi/2.$$

Using the initial conditions, we obtain

$$A_n = \frac{2}{l}\int_0^l (U_c - U_0)\cos\frac{(2n-1)\pi}{2}\frac{x}{l}\,dx,$$

$$B_n = \frac{h}{k}A_n - \frac{2}{l}\int_0^l \varphi_2(x)\cos\frac{(2n-1)\pi}{2}\frac{x}{l}\,dx.$$

Relation (13) has been verified experimentally by an investigation of capillary absorption of liquid by capillary-porous substances. The substances used were sand and clay (see table).

The experimental technique was as follows. The material under examination (sand) was poured into a column with a mesh bottom. The column was made up of glass sections, cemented together with BF-2 cement, the ends of the sections being carefully ground to obtain good contact. For the tests with clay, cylindrical specimens were prepared, dried at room temperature, and then in a drying oven at 100°–105° C. The column containing sand (or the clay cylinder) and an attached scale were brought into contact with water, and the height of the capillary rise of the liquid was noted on the scale.

The experiment showed the relationship between height of rise and time. Using the data obtained the moisture content of the system at various heights of capillary rise was calculated from (13). In addition, at the end of the test, the column containing sand was dismantled, and the moisture content of each section was measured by the usual method of drying in weighing bottles. Values of the moisture content as calculated from (13) and as obtained by experiment are shown graphically in Figs. 1 and 2.

The moisture contents calculated from the formula which does not take into account relaxation phenomena are quite close to the experimental values at the beginning of the capillary rise, but as the rate of absorption decreases, the divergence increases, and the theoretical moisture contents become far greater than those obtained by experiment. Therefore, as may also be seen from an analysis of Eq. (2), the flow of liquid at small velocities in a capillary-porous material is mainly due to relaxation of the elastic stresses in the liquid. When relaxation phenomena are allowed for, there is satisfactory agreement between the theoretical and the experimental values of moisture content.

NOTATION

i) mass flux; D) diffusion coefficient; ρ) concentration; n) normal to the isoconcentration surface; τ_r) relaxation time; w) rate of shift of isoconcentration surface; η) coefficient of viscosity; ε) relative strain of body; P) shear stress; G) modulus of elasticity; τ) time; x, y, z) rectangular Cartesian coordinates; U) moisture content of material; l) linear dimension; U_c) saturation moisture content; γ_0) specific weight of absolutely dry material in unit volume of moist substance.

REFERENCES

1. A. V. Luikov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat, 1963.

2. A. V. Luikov, Theorerical Foundations of Building Thermophysics, [in Russian], Izd. AN BSSR, 1961.

3. M. Reines, Lectures on Theorerical Rheology [Russian translation], vol. 1, IL, 1962.

4. N. S. Koshlyakov et al., Differential Equations of Mathematical Physics [in Russian], Fizmatgiz, 1962.

5. A. V. Luikov, IFZh, 9, no. 3, 1965.

17 June 1965
Institute of Heat and Mass Transfer
AS USSR, Minsk